Selecting Interventions for Children Who Struggle with Mathematics

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Presented to the Texas Association of School Psychologists

What is Mathematics?

- Has practical and theoretical uses
  - Universal and utilitarian subject
  - Esoteric and esthetic subject
    - Expertise in advanced mathematics is obtained by only a small subgroup of society
    - Helps people understand nature and control their environments
      - Math is used for developing and testing theories
      - Math is used by societies to manage endeavors and guide allocation of goods and services

"In God we trust; all others bring data."
Dr. W. Edwards Deming, American statistician

Thanks

- Much of this presentation has been created by three other collaborators – Dr. Nicholas Benson of the University of South Dakota, and Kayla Kolander and Karen Wall who are currently on internship.

How Important is Mathematics to Our Children's Future?

- The National Science Board (2008) reported that growth of jobs in the mathematics-intensive science and engineering workforce is outpacing overall job growth by 3:1.

Presentation Objectives

- Define Mathematics: Why is it important?
- Early Math Skills
  - Assessment
  - Instruction
  - Intervention
- Elementary
  - Assessment
  - Instruction
  - Intervention
- Middle School
  - Assessment
  - Instruction
  - Intervention
- Assessment and intervention for difficulties with algebra
- Describe productive dispositions
- How to distinguish difficulties from disorders

How are US students doing? 2009 NAEP Data

- For 8th grade students, 68% of students performed below the proficient level across the U.S.
- For 8th grade students, 29% performed below basic.
- For 4th graders, 61% below proficient.
- For 4th graders, 19% below basic.

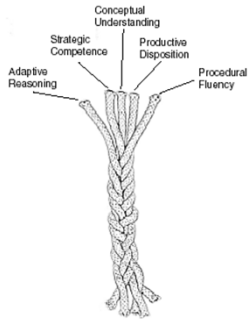
nces.ed.gov

How are US students doing? Trends in International Mathematics and Science Study

- American 4th and 8th grade performance below international average in Knowing Facts & Procedures and Math Reasoning (Ginsburg et al., 2005).

Retrieved from http://www.air.org/news/documents/TIMSS_PISA%20math%20study.pdf

National Research Council's (2001) Intertwined Strands of Mathematical Proficiency



Change is Needed!

- Without substantial and sustained changes to its educational system, the United States will relinquish its leadership in the 21st century… it is yet more fundamental to recognize that the safety of the nation and the quality of life—not just the prosperity of the nation—are at issue (National Mathematics Advisory Panel, 2008).

Intertwined Strands of Mathematical Proficiency

- ***Conceptual understanding***
  - –Comprehension of mathematics concepts, operations and relations
- ***Procedural fluency***
  - –Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic competence***
  - –Ability to formulate, represent, and solve mathematical problems
- ***Adaptive reasoning***
  - –Capacity for logical thought, reflection, explanation, and justification
- ***Productive disposition***
  - –Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Kilpatrick, Swafford, & Findell (2001)

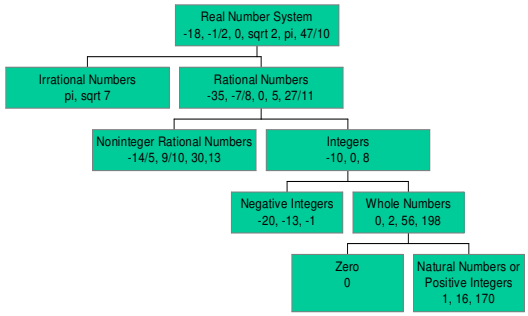
How do students acquire math proficiency?

- It is crucial for educators (and preferably parents as well) to understand and possess math proficiency.
- Understanding math proficiency
  - – National Research Council's intertwined strands of mathematical proficiency

Abstraction

- Mathematics involves the use of abstractions
- Numbers and operations are abstractions based on experience but independent of particular experiences
- Number systems allow for simultaneous consideration of numbers and operations

Real Number System



Math as Metaphors

- When choosing and translating among representations learners must balance the following characteristics:
  - transparency
  - efficiency
  - generality
  - clarity
  - precision

Math as Metaphors

- Mathematical representations are suggestive of corresponding metaphors
- Learners use multiple metaphors when developing meaningful mathematical concepts
- Types of representations
  - **Visual images**
  - **Notation**
  - **Kinetic understanding**
  - **Metaphorical understanding**

Math Building Blocks

- Number
- Shape
- Spatial relationships
- Change
- Chance
- These building blocks have connections and dependencies

(Kilpatrick, Swafford, & Findell , 2001)

Math as Metaphors

- Experts begin to ***understand the concept in its own terms*** and the images and metaphors tend to ***fade away from awareness…***
- Why?

Number Sense

- Definition of number sense is unclear and has varied across researchers
- Characteristics of number sense described by Kalchman, Moss & Case (2001) include:
  - Fluency in estimating and judging magnitude
  - Ability to recognize unreasonable results
  - Flexibility when mentally computing
  - Ability to move among different representations and select the most appropriate one

Big Ideas of Early Math

(Clements, Sarama, & DiBiase, 2004)

- Counting
- Comparing and ordering
- Equal partitioning
- Composing and decomposing
- Grouping and place value
- Adding to/taking away

Assessment of Early Math Skills

Assessment Name	Type	Reliability	Validity
AIMSweb Test of Early Numeracy (TEN)	Curriculum-based measure	0.76-0.99 *See table on following slide	0.60-0.75
Preschool Numeracy Indicators (PNIs)	Curriculum-based measure	0.62-0.96 Varies amongst individual measures	
TEMA (Test of Early Mathematics Ability)	Norm-referenced measure	0.90	0.55-0.65

www.AIMSweb.com
Mental Measurements Yearbook

ASSESSMENT OF EARLY MATH SKILLS

AIMSweb Early Mathematics Curriculum-Based Measurement (EM-CBM)

Reliability For All Testing Sessions

EM-CBM Measure	Inter-Scorer	Alternate-Form		Test-Retest	
		Fall$_a$	Winter$_b$	13wks$_a$	26wks$_b$
Oral Counting	.99	-	.93	.80	.78
Number Identification	.99	.89	.93	.85	.76
Quantity Discrimination	.99	.93	.92	.85	.86
Missing Number	.98	.83	.78	.79	.81

Note. Test-retest reliabilities based on Form A. an = 12, bn = 52.

http://www.aimsweb.com/uploads/news/id34/06.clarkeshinn.pdf

Curriculum Based Assessments

- General outcome measure
- Set of skills based on curriculum
- Purposes
- Decisions

(Hosp, 2009)

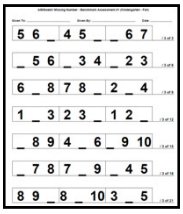
(Shinn, 1998)

AIMSweb Tests of Early Numeracy

Administration Time

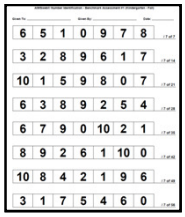
Measure	Timing	Test Arrangements	What is Scored
Oral Counting (OC)	1 minute	Individual	Correct Oral Counts
Number Identification (NI)	1 minute	Individual	Correct Number Identifications
Quantity Discrimination (QD)	1 minute	Individual	Correct Quantity Discriminations
Missing Number (MN)	1 minute	Individual	Correct Missing Numbers

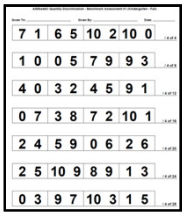
AIMSweb Tests of Early Numeracy



Quantity Discrimination →

← Missing Number





Number Identification →

And…
Oral Counting

www.AIMSweb.com

What about Shape, Spatial relationships, and Chance

- How might these be integrated into instruction?
- Should these skills be assessed?

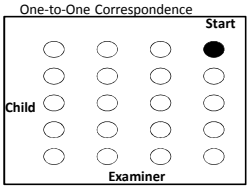
Preschool Numeracy Indicators

Quantity Comparison

Sample 1: ___ 6 : 2
Sample 2: ___ 4 : 5

1. ___ 4 : 1	11. ___ 3 : 4	21. ___ 5 : 1
2. ___ 1 : 5	12. ___ 2 : 1	22. ___ 6 : 2
3. ___ 2 : 5	13. ___ 1 : 4	23. ___ 4 : 6
4. ___ 3 : 6	14. ___ 5 : 4	24. ___ 4 : 5
5. ___ 6 : 4	15. ___ 4 : 3	25. ___ 2 : 4
6. ___ 2 : 6	16. ___ 3 : 5	26. ___ 5 : 6
7. ___ 6 : 5	17. ___ 5 : 3	27. ___ 3 : 2
8. ___ 4 : 2	18. ___ 1 : 6	28. ___ 6 : 3
9. ___ 1 : 3	19. ___ 3 : 1	29. ___ 6 : 1
10. ___ 1 : 2	20. ___ 2 : 3	30. ___ 5 : 2

One-to-One Correspondence



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Oral Counting Fluency

3	6	5	2	9	15	19	8	18	7
11	1	12	4	13	14	20	16	10	0
17	20	2	18	8	14	13	3	9	5
19	11	17	4	7	6	10	0	12	15
1	16	3	19	4	18	12	6	9	16
1	2	20	7	14	17	8	0	10	15
13	11	5							

Number-Naming Fluency

(Floyd & Hojnoski, 2005)

EARLY MATH INSTRUCTION AND INTERVENTION

Test of Early Mathematics Ability

- TEMA-3
  - Measures math performance for children ages 3-0 to 8-11
- Two parallel forms measure:
  - Numbering skills
  - Number-comparison facility
  - Numeral literacy
  - Mastery of number facts
  - Calculation skills
  - Understanding concepts

Evidence Based Practice for Early Math

- Information from assessments should guide your choice of intervention
- Focus on interventions that are supported by research
- Training of all persons involved in implementing the intervention is crucial to its fidelity
- Instruction during the intervention should be **systematic and explicit.**

(Burns, VanDerHeyden & Boice, 2003) (Kratochwill &Shernoff, 2003)

Preschool Interventions

Intervention Name	Type/Modality	Effect Size	Target Population
Building Blocks for Math	Computer assisted, manipulatives, and print materials	1.06	Individual, small group or large group
Pre-K Mathematics	Traditional teacher-guided	0.58	Small group

What Works Clearinghouse
http://ies.ed.gov/ncee/wwc/

ASSESSMENT DURING THE ELEMENTARY YEARS

Building Blocks for Math

- "Mathematize" every day activities
- Individual, small, and large group activities
- Embeds math learning into daily activities
- uses *Building Blocks for Math PreK software,* manipulatives, and print material

What Works Clearinghouse
http://ies.ed.gov/ncee/wwc/

When performance is low

- Determine if there has been a lack of opportunity for the child to learn mathematics
  - Consider the family situation
  - Consider educational history
  - Consider strategies that have been attempted to address problems
    - Did these strategies target the appropriate skills?
    - Were these strategies implemented accurately and with appropriate intensity?

Pre-K Mathematics

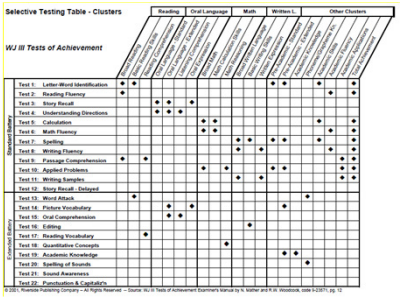
- Organized into seven units to teach informal math knowledge to preschool children
  - (1) Counting and Number,
  - (2) Understanding Arithmetic Operations (Fall Activities),
  - (3) Spatial Sense and Geometry,
  - (4) Patterns,
  - (5) Understanding Arithmetic Operations (Spring Activities),
  - (6) Measurement and Data, and
  - (7) Logical Reasoning

What Works Clearinghouse
http://ies.ed.gov/ncee/wwc/

Standardized Assessments

- Woodcock Johnson Test of Achievement-III
- Wechsler Individualized Achievement Test-III
- Early Mathematics Diagnostic Assessment (EMDA)

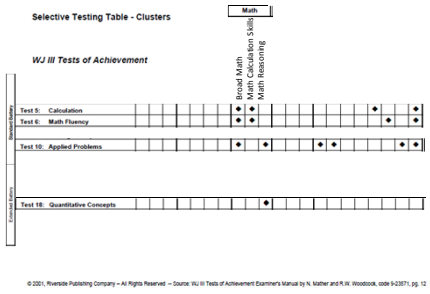
Woodcock-Johnson test of Achievement- 3rd Edition (WJIII)



Wechsler Individual Achievement Test- 3rd Edition (WIAT-III)

- PreK-12th grade
- Subtests:
  - Math Problem Solving
  - Numerical Operations
  - Math Fluency
    - Addition
    - Subtraction
    - Multiplication

Woodcock-Johnson test of Achievement- 3rd Edition (WJIII)



Assessment

Domain to Be Assessed	Screening Tools	Progress Monitoring	Diagnostic Measures
Number Sense	AIMSweb CBM	AIMSweb CBM	EMDA
Operations	AIMSweb CBM	AIMSweb CBM	WJ-III EMDA
Patterns & Relationships	AIMSweb CBM	AIMSweb CBM	EMDA
Measurement	AIMSweb CBM	AIMSweb CBM	EMDA
Geometry	AIMSweb CBM	AIMSweb CBM	EMDA
Data and Probability	AIMSweb CBM	AIMSweb CBM	EMDA

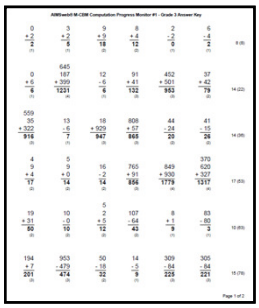
(AIMSweb, 2008) (Riverside Publishing, 2009)
(Pearson Assessment, 2005) (Pearson, 2009)

WJ-III Interpretive Information

Qualitative
- Behavioral observations during testing
  - Attitude
  - Motivation
  - Level of engagement
  - Persistence/effort
  - Response style
- Error analysis of responses to individual items
  - Patterns
  - Identify strengths/weaknesses
- Level of attention/concentration

Mather, Wendling & Woodcock, 2001

AIMSweb Math Curriculum-Based Measurement (M-CBM)



Early Mathematics Diagnostic Assessment (EMDA)

- Formative Assessment
- Pre-K to 3rd Grade
- 15 to 20 minutes
- Designates one of 3 performance levels

(Pearson Assessment, 2005)

Evidence Based Practice for Early Math

- Domains
  - Number Sense
  - Calculation
  - Patterns and relationships
  - Measurement
  - Geometry
  - Data and Probability

(AIMSweb, 2008)

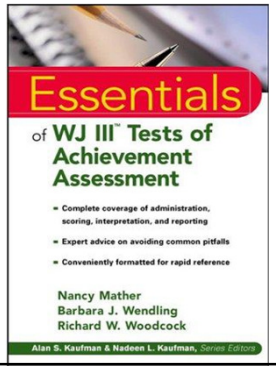
Functional Assessment of Math Skills

- 5 Main Reasons Children Struggle
  - 1. They do not want to do it
  - 2. They have not spent enough time doing it
  - 3. They have not had enough help to do it
  - 4. They have not had to do it that way before
  - 5. It is too difficult

(Daly & Martens, 1997)

Linking WJ-III Results to Interventions

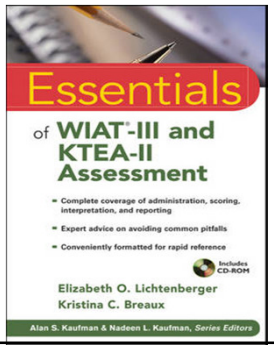
- Calculation
- Math Fluency
- Applied Problems
- Quantitative Concepts



INSTRUCTION AND INTERVENTION DURING THE ELEMENTARY YEARS

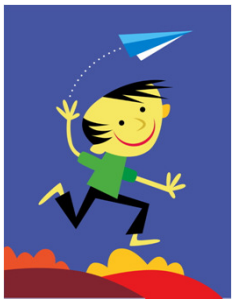
Linking WIAT-III Results to Interventions

- Look at specific skills deficit
- Look at strategies employed during testing
- Compare Numerical Operations and Math Problem Solving
- Compare Numerical Operations and Fluency subtests



Linking Functional Analysis Results to Interventions

- No motivation
- Insufficient prompting and feedback
- Poor accuracy in target skill
- Poor fluency in target skill
- Does not generalize the use of the skill

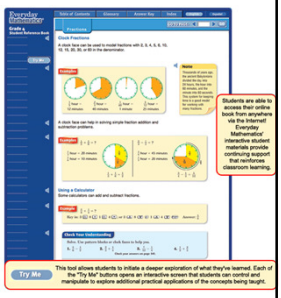


(Daly & Martens, 1997)

Mathematics Curricula-NSF Funded

- Everyday Mathematics
  - Designed for students pre K-6
  - Covers a variety of domains

For More Information See:
https://www.everydaymathonline.com/
(Woodward & Baxter, 1997)
(SRA/McGraw Hill, 2003)
(Riordan & Noyce, 2001)
(Waite, 2000)



(Wright Group, 2007)

(Slavin & Lake, 2007)

Modality of Interventions

- Mathematics Curriculum
  - National Science Foundation (NSF) Funded
  - Saxon Math
  - Traditional commercial textbook programs
- Computer-Assisted Instruction (CAI)
  - Computer Managed Learning Systems
- Instructional Process Programs
  - Cooperative Learning
  - Cooperative/Individualized Programs
  - Direct Instruction
  - Mastery Learning
  - Professional Development Focused on Math Content
  - Professional Development Focused on Classroom Management and Motivation
  - Supplemental Programs

(Slavin & Lake, 2007)

Mathematics Curricula-Traditional

Intervention	Effect Size	Best Results
Scott Foresman-Addison Wesley	-0.01 to +0.04	-Grades 3rd to 5th -Computation
Houghton Mifflin Mathematics	+0.14	-Grades 2nd to 5th
Growing with Mathematics	+0.22	-Calculation -Comprehension
Excel Math	+0.13	-Grades 2nd to 4th
MathSteps	+0.03	-3rd Grade
Knowing Mathematics	+ 0.10	-Calculation

(Slavin & Lake, 2007)

Mathematics Curricula-NSF Funded

Intervention	Effect Size	Best Results
Everyday Mathematics	-0.25 to +0.26	-Measurement -Calculation -Geometry -Students who used EM for 2+ Years -Grades 3rd to 5th
Math Trailblazers	+0.06	Concepts

(Slavin & Lake, 2007)

Mathematics Curricula-Traditional

- Growing with Mathematics
  - Pre-K through 5th curriculum
  - Activity based, problem solving approach
  - Real world connections
  - Major focus is number sense

For More Information See:
https://www.wrightgroup.com/

(Slavin & Lake, 2007)

Computer Assisted Instruction

Intervention	Effect Size	Best Results
Compass Learning (Jostens)	-0.08 to +0.66	-Grades 2nd and 3rd
Success Maker	-0.06 to +0.36	-Calculation -3rd Grade
Classworks	+0.21 to +0.85	-3rd Grade
Accelerated Math	+0.03 to +0.23	-Grades 3rd to 5th

(Slavin & Lake, 2007)

Cooperative/Individualized Programs

Intervention	Effect Size	Best Results
Team Assisted Individualization (TAI)	-0.11 to +0.38	-Calculation -Grades 3rd to 6th
Project CHILD	+0.69	-Grades 2nd to 5th

(Slavin & Lake, 2007)

Computer Assisted Instruction

- Classworks
  - K through 12 system
  - Each unit helps students learn a specific concept by offering a variety of activities
  - Product Demo: http://classworks.com/video/demo_wrapper.php

For More Information See:
http://www.classworks.com/
(Patterson, 2005)
(Whitaker, 2005)

(Slavin & Lake, 2007)

Direct Instruction

- Connecting Math Concepts
  - Grades K-3
  - Effect Sizes: +0.26-+1.33
  - Best Results: Calculation and Concepts
  1. Big Ideas
  2. Prerequisite Skills
  3. Explicit Instruction
  4. Guided Practice
  5. Connection of new strategies to prior knowledge
  6. Cumulative review

For More Information See:
(Crawford & Snider, 2000)

(Slavin & Lake, 2007)

Cooperative Learning

Intervention	Effect Size	Best Results
Classwide Peer Tutoring	+0.33	Grades 1st to 4th
Peer Assisted Learning Strategies	+0.10 to +0.24	-Students classified as average achieving and students with disabilities showed the best results

(Slavin & Lake, 2007)

Mastery of Learning

- Topic for 2 weeks
- Formative test
- Students who pass with 80% accuracy move on to enrichment activities
- Overall Effect Size ranges from -0.18 to +1.08
- Best Results: Calculation

(Slavin & Lake, 2007)

Professional Development Focused on Math Content

Intervention	Effect Size	Best Results
Cognitively Guided Instruction	+0.24	-Computation-
Dynamic Pedagogy	+0.32	-3rd Grade

(Slavin & Lake, 2007)

"Small Group Tutoring"

- 3rd Grade
- Hot Math

Fuchs et al., 2008

Professional Development Focused on Classroom Management and Motivation

Intervention	Effect Size	Best Results
Missouri Mathematics Project	+0.33 to +0.57	-Grades 2nd to 4th
Consistency Management and Cooperative Discipline	+0.33 to +0.53	-3rd Grade

(Slavin & Lake, 2007)

Elementary Math Conclusions

- Link assessment or functional analysis results to intervention
- More domain-specific interventions needed

Supplemental Programs

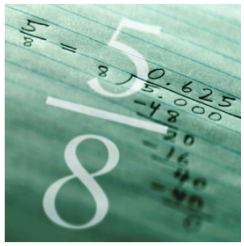
Intervention	Effect Size	Best Results
Small Group Tutoring	+0.37	-Calculations (Addition) -Concepts -Story Problems
Every Day Counts	+0.15	-5th Grade
Project SEED	+0.25 to +0.73	-Concepts

(Slavin & Lake, 2007)

ASSESSMENT DURING THE MIDDLE SCHOOL YEARS

Assessment During the Middle School Years

- Focus
  - Fractions
  - Decimals
  - Percent



(National Math Advisory Panel)

Characteristics of Students with Low Math Achievement

Low Math Reasoning Skills:

- Limited math vocabulary
- Lacks age-appropriate quantitative concepts
- Trouble with estimation
- Limited strategies for solving math problems
- Does not recognize or self-correct errors
- Difficulty recognizing relevant information in word problems
- Difficulty eliminating extraneous information from word problems

Mather, Wendling & Woodcock, 2001

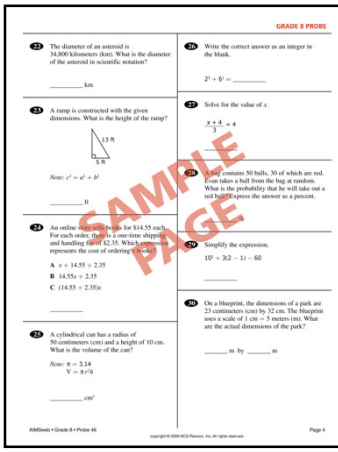
Middle School Assessments

Assessment Name	Type	Reliability	Validity
Woodcock-Johnson Tests of Achievement -Math Calculation Skills -Math Reasoning	Norm-referenced measure	~0.90*	0.50-0.80
AIMSweb Math (computation/facts)	Curriculum-based measure	0.66-0.97	0.43-0.54**

http://web.ebscohost.com.ezproxy.usd.edu/ehost/detail?vid=3&hid=2&sid=762860d7-723f-4b43-b45d-a0e0d48f40c6%40sessionmgr11&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#db=loh&AN=15072743

www.AIMSweb.com

INSTRUCTION AND INTERVENTION DURING THE MIDDLE SCHOOL YEARS



Concepts and Applications (M-CAP)

Get more information from AIMSWEb site

Instructional Implications of Mathematics Assessment for Students with Low Math Achievement

- Instructional level
- Environment
- Manipulatives
- Number of problems
- Additional time
- Calculator
- Graph paper
- Practice
- Monitor Performance
- Fact charts
- Meaningful applications
- Vocabulary
- Functional math

Mather, Wendling & Woodcock, 2001

Middle School Interventions

Intervention Name	Type/Modality	Effect Size	Target Population
Saxon Middle School Math	Traditional teacher-guided, text-based, manipulatives (Curriculum)	0.41	Large group (grades 6-9)
Expert Mathematician	Integrated computer software and workbook activities (Supplemental)	0.35	Pairs of students as large group, small group or more individualized
Cognitive Tutor	Computer Assisted Instruction (Supplemental), cooperative teams, traditional teacher instruction	0.38	Large group (grade 9)

What Works Clearinghouse http://ies.ed.gov/ncee/wwc/
Slavin, Lake, Groff, 2008

Cognitive Tutor

- Known by other names:
  - Carnegie Algebra Tutor
  - Pittsburgh Urban Mathematics Project (PUMP)
- Supplemental Computer Assisted Instruction
  - Tutoring system emphasizing algebra problem-solving
- Students carry out investigations of real-world problems
  - Use spreadsheets, graphers, and symbolic calculators

Slavin, Lake, Groff, 2008

Saxon Middle School Math

- Whole class, traditional textbook-based Math intervention
  - Courses 1-3 (Grades 6-8)

Additional Middle School Interventions

Intervention Name	Type/Modality	Effect Size	Target Population
IMPROVE	Cooperative learning, metacognitive instruction, mastery learning (Alternative to ability grouping)	0.52	Large group, small groups
Metacognitive Strategy Instruction	Cooperative learning	0.67	Small groups

Slavin, Lake, Groff, 2008

The Expert Mathematician

- Students work in pairs
  - Active, interdependent sessions
  - Roles: Keyboarder and Reader-Recorder
- Flexible
  - Useful for At-Risk students as well as students needing more challenging material
- Engaging
  - Keeps students on task

http://www.expertmath.org/
Slavin, Lake, Groff, 2008

IMPROVE

I introducing new concepts
M metacognitive questioning
P practicing
R reviewing and reducing difficulties
O obtaining mastery
V verification
E enrichment

Slavin, Lake, Groff, 2008

IMPROVE

- Alternative to ability grouping
  - Accommodates diversity in heterogeneous class
  - Students work in small groups
- Teacher introduces concepts to class
- Students work in groups
  - Ask/answer metacognitive questions
- After 10 lessons, take formative test on unit content

Slavin, Lake, Groff, 2008

Algebra

- Algebra is a gateway for more advanced math (National Mathematics Advisory Panel)
- Fluency with whole numbers, fluency with fractions, and fluency with beginning geometry skills are the critical foundations of algebra.
- The sharp falloff in mathematics achievement in the U.S. begins as students reach late middle school, at which time U.S. students struggle with algebra (TIMMS).

Metacognitive Strategy Instruction

- Key component for IMPROVE
- Self-regulated learning
- Work in small groups
  - Ask questions aloud over comprehension, connections, similarities/differences with other problems, appropriate strategies, and reflection

Slavin, Lake, Groff, 2008

Probe samples from http://www.ci.hs.iastate.edu/aaims/

ALGEBRA PROBES (FOEGEN, 2008)

ASSESSMENT AND INTERVENTION FOR DIFFICULTIES WITH ALGEBRA

Basic Concepts

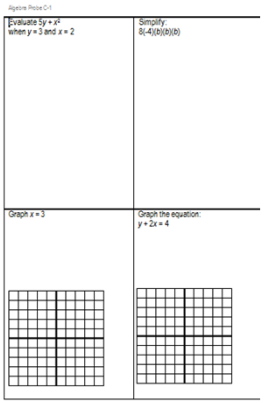
- 5 minutes
- 1 point per correct response

Algebra Probe A-1

Solve: $9 + a = 15$	$a =$
Evaluate: $12 + (-8) + 3$	
Simplify: $2x + 4 + 3x + 5$	
Solve: $12 - e = 4$	$e =$
Simplify: $4(3 + s) - 7$	
Simplify: $b + b + 2b$	

Content Analysis

- 3 points per problem
- Rubric for scoring
- 7 minutes



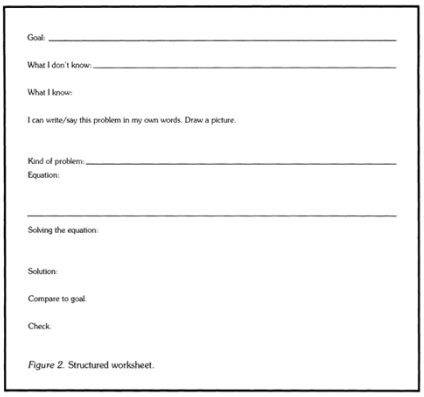


Figure 2. Structured worksheet.

Algebra Foundations

- 5 minutes
- 1 point per response

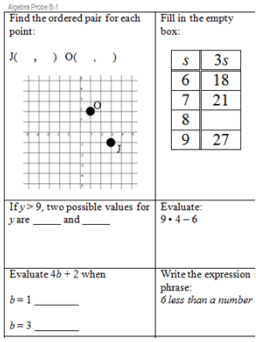


Table 7
Summary of Proportion of Comparison and Instructed Students Reaching Criterion on Instructed Problems at Pretest and Posttest and Fisher's Exact Test

Problem Type	Dependent Variable	Pretest C	Pretest I	Posttest C	Posttest I	Sig. at Post-
Relational	Representation	0 / 8	0 / 12	0 / 8	12 / 12	<.05
Relational	Solution	0 / 8	0 / 12	0 / 8	11 / 12	<.05
Relational	Answer	0 / 8	0 / 12	0 / 8	12 / 12	<.05
Proportion	Representation	0 / 8	0 / 12	0 / 8	10 / 12	<.05
Proportion	Solution	0 / 8	0 / 12	1 / 8	11 / 12	<.05
Proportion	Answer	3 / 8	2 / 12	3 / 8	11 / 12	<.05
Two-Variable Two-Equation	Representation	0 / 8	0 / 12	0 / 8	6 / 12	<.05
Two-Variable Two-Equation	Solution	0 / 8	0 / 12	0 / 8	6 / 12	<.05
Two-Variable Two-Equation	Answer	0 / 8	0 / 12	0 / 8	6 / 12	<.05

Cognitive Strategy

Self-Questions for Representing Algebra Word Problems

1. Have I read and understood each sentence? Are there any words whose meaning I have to ask?
2. Have I got the whole picture, a representation, for this problem?
3. Have I written down my representation on the worksheet?
   (goal; unknown(s); known(s); type of problem; equation)
4. What should I look for in a new problem to see if it is the same kind of problem?

Self-Questions for Solving Algebra Word Problems

1. Have I written an equation?
2. Have I expanded the terms?
3. Have I written out the steps of my solution on the worksheet? (collected like terms; isolated unknown(s); solved for unknown(s); checked my answer with the goal; highlighted my answer)
4. What should I look for in a new problem to see if it is the same kind of problem?

Figure 1. Self-questions on prompt cards.

Hutchinson, 1993

Table 1
Algebra Progress Monitoring Measures

	Basic Skills	Algebra Foundations	Translations	Content Analysis
Conceptual Underpinnings	Automaticity; skills in which proficient students should be fluent	Core concepts in algebra and pre-algebra	Minimal symbolic manipulation; ability to move fluently between multiple representational modes	Sampling of key concepts and skills from a traditional Algebra 1 text; assesses maintenance and generalization
Type of Task	Production	Production	Selection (4 item multiple choice)	Selection (4-item multiple choice) with option to show work
Number of Items/Duration	60/5 minutes	50/5 minutes	42/7 minutes	16/7 minutes
Scoring	1 point per response/correct or incorrect	1 point per response/correct or incorrect	1 point per response; penalty for guessing	3 points per problem; rubric for partial credit

Note. The Basic Skills, Algebra Foundations, and Translations measures all represent indicators of general proficiency in algebra; the Content Analysis measure reflects key skills and concepts in the instructional curriculum.

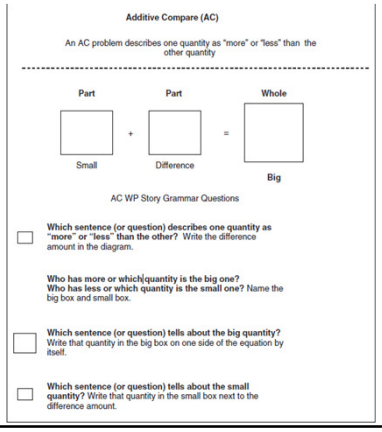
Foegen, 2008, p. 67

CRA - CSA

- Concrete – Represenational (Semi-Concrete)-Abstract.
- Graduated teaching sequence.
  - Concrete (e.g. objects)
  - Semi-concrete (e.g. drawings)
  - Abstract (symbols)
- STAR (Search, Translate, Answer, Review)

(Foegen, 2008)

Xin et al, 2008

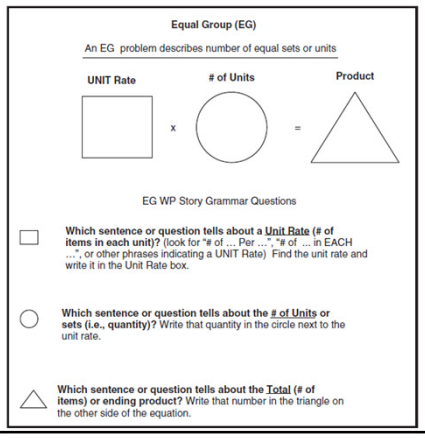


Story Grammar (Xin, Wiles, & Lin 2008)

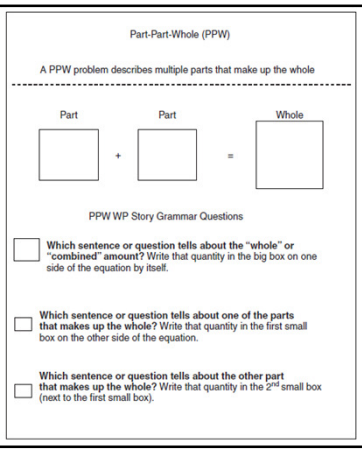
Table 2a
Sample Problems in Criterion Tests: Addition and Subtraction

Problem Type	Sample Problem Situation
Part-Part-Whole	
	Combine
Part (or smaller group) unknown	Jamie and Daniella have found out that together they have 92 books. Jamie says that he has 57 books. How many books does Daniella have?
Whole (or larger group) unknown	Victor has 51 rocks in his rock collection. His friend, Maria, has 63 rocks in her collection. How many rocks do the two have altogether?
	Change-Join
Part (or smaller group) unknown	Luis had 73 candy bars. Then, another student, Lucas, gave him some more candy bars. Now he has 122 candy bars. How many candy bars did Lucas give Luis?
	A girl named Selina had several comic books. Then, her brother Andy gave her 40 more comic books. Now Selina has 67 comic books. How many comic books did Selina have in the beginning?
Whole (or larger group) unknown	A basketball player ran 17 laps around the court before practice. The coach told her to run 24 more at the end of practice. How many laps did the basketball player run in total that day?
	Change-Separate
Part (or smaller group) unknown	Davis had 62 toy army men. Then, one day he lost 29 of them. How many toy army men does Davis have now?
	Ariel had 141 worms in a bucket for her big fishing trip. She used many of them on the first day of her trip. The second day she had only 68 worms left. How many worms did Ariel use on the first day?
Whole (or larger group) unknown	Alexandra had many dolls. Then, she gave away 66 of her dolls to her little sister. Now, Alexandra has 63 dolls. How many dolls did Alexandra have in the beginning?

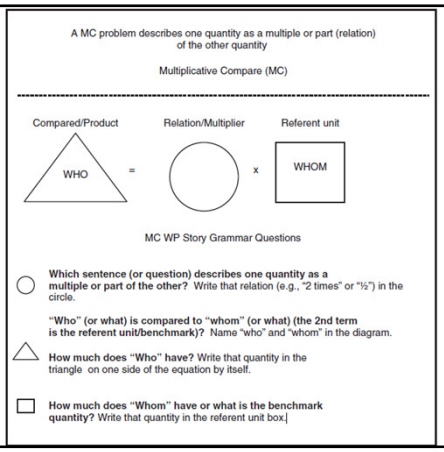
Xin et al, 2008



Xin et al, 2008, p. 171



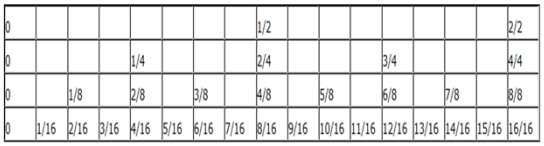
Xin et al, 2008



Xin et al., 2008, p. 172

- "Detect the problem type.
- Organize the information using WP conceptual model (e.g., Part + Part = Whole) diagrams.
- Transform the diagram into a meaningful math equation.
- Solve for the unknown quantity or variable in the equation and check your answer."

Graphic Organizers (Fractions)



Source: K8accesscenter.org

ALGEBRA INTERVENTIONS

Mayfield & Glenn, 2008

- Teaching more abstract algebra problems
- Cumulative Practice
  - Practice the algebra and then give praise for correct and correction for incorrect
- Tiered Feedback
  - Tier 1 – Just mention how many problems are incorrect.'
  - Tier 2 – Highlight correct problems
  - Tier 3 – Identify a consistent error pattern and point it out.

Graphic Organizers

- I need to look up some specific interventions for graphic organizers.

Mayfield & Glenn, 2008

- Feedback + Solution Sequence: Explain the wrong items – show the participant how to perform the task.

- Review Practice
  - Review a target skill necessary to solve the algebra problem

PRODUCTIVE DISPOSITIONS

Further suggestions

- 1) Continue to instruct in basic arithmetic
- 2)Use think aloud techniques
- 3) Guided practice before independent practice
- 4) Physical & pictorial models
- 5) Relate to real life interests

Source: Witzell, Smith, & Brownell, 2001

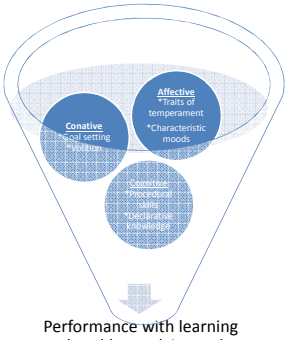
Assessment of Productive Dispositions

- Effort counts!
  - (National Mathematics Advisory Panel, 2008)
- Assessment of:
  - Temperament
  - Affect
  - Self-regulation
  - Attitudes (pertaining to education, math, etc.)
  - Motivation (proximal and distal), goal-setting (learning goals, Grant & Dweck, 2003), and volition

Complex Mathematical Skills that May be Impacted by Various Learning Disabilities and/or Cognitive Weaknesses

- Algebra
  - Representation and transformation
- Geometry
  - Measuring length and area
  - Shape and form
- Calculus
  - Study of change
- Differential Equations
  - Modeling real-word problems to approximate solutions
- Statistics and Probability
  - Description
  - Inferences

Richard Snow's Cognitive-Affective-Conative Triad



The Role of Affect in Cognitive Processing

- Mood and related emotional states direct attention with good or bad effects on learning
- Positive affect results in systematic, meaningful organization of information
- Anxiety likely yields unproductive thoughts that intrude on task performance

(Schwarz & Bohner, 1996)

Dispositions Likely to Improve Math Performance and Achievement

- Before seventh grade, and in easier math courses, self-standards seem to differentiate achievement groups
- Achievement groups are differentiated by strategy selection and motivational processes in rigorous math classes-and use of strategy selection and motivational processes is related to interest in and perceived value of math tasks

(Cleary & Chen, 2009)

The Role of Conative Process in Cognitive Processing

- Motivation
  - Task interest
  - Perceived task value
  - Goal setting
- Volition
  - Action orientation
    - Focus on action limits increased negative affect and decreased future performance in response to past failures
  - Action controls
    - Strategies for maintaining intentions (e.g., choosing supportive environments, scheduling, prioritizing tasks, self-reinforcement)
  - Mindfulness
    - Seek opportunities for investing mental effort
    - Deliberate application of learning strategies
    - Set high self standards when reflecting on their performance
- Both motivation and volition require self-regulation

Classroom Variables Likely to Improve Math Performance and Achievement

- Making math intrinsically interesting related to becoming strategically involved in learning (Midgley & Edelin, 1998; Reeve & Jang, 2006; Urdan & Midgley, 2003)
- Increasing choice and autonomy may help improve student engagement (Dweck & Leggett, 1988; Midgley & Edelin, 1998)
- Some students may need to be taught self-regulation strategies (Cleary & Chen, 2009)

Self-regulation

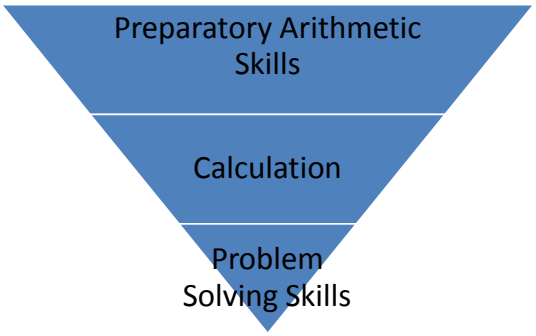
- Thoughts, feelings, and behaviors that are proactive, goal-oriented, and modified in response to internal or external performance feedback (Zimmerman, 1989)
- Involves forethought, performance control, and reflection
- Students may self-regulate more effectively in some situations but not others (Hadwin et al., 2001)

DISTINGUISHING DIFFICULTIES FROM DISORDERS

Distinguishing difficulties from disorders

- How do we differentiate students with math difficulties from those with disabilities?
  - Unresponsiveness to intervention
  - Neuropsychological profiles (e.g., Hale & Fiorello, 2004)
  - Process assessment (e.g., Bryant & Rivera, 1997)
  - Other signs and symptoms

Domains of Math Interventions



Mathematics Disorder

- Difficulties with number sense and, to a greater degree, calculation are hallmarks of MD
- Impairments with problem solving also are likely
  - The inability to carry out calculations historically has been referred to as dyscalculia
- Research suggests there are neurological substrates of number sense and calculation

When choosing and organizing an intervention, keep in mind…

1. The method used to teach students mathematics
   - Both self-instruction and direct instruction seem to be adequate methods for students with special needs.
   - For the learning of basic math facts, direct instruction appears to be most effective.
   - For the learning of problem-solving skills, self-instruction methods are also quite effective.
2. A second finding concerns the use of CAI, which can be very helpful when students have to be motivated to practice with certain kinds of problems.
   - possible to let children practice and automatize math facts and also to provide direct feedback (e.g., Koscinski & Gast, 1993).
   - However, the computer cannot remediate the basic difficulties that the children encounter.
   - The results of the present study show that in general, traditional interventions with humans as teachers, and not computers, are most effective.
3. We often have children work together in order that they might help and teach each other. It appears, however, that children with special needs do not particularly profit from this strategy.
   - Of course, peer tutoring may be helpful and effective at times, but the present study shows that it cannot replace or be as effective as instruction by an adult teacher.
4. Finally, not all of the changes proposed by math reformers are as effective as more traditional approaches.
   - However, it always takes time to adjust to changes, and this variable should therefore be re-examined thoroughly when more data become available.

Kroesbergen & Van Luit, 2003.

CONCLUSIONS AND DISCUSSION

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